Probabilistic Automata for Safety LTL Specifications

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joint work with Mahesh Viswanathan

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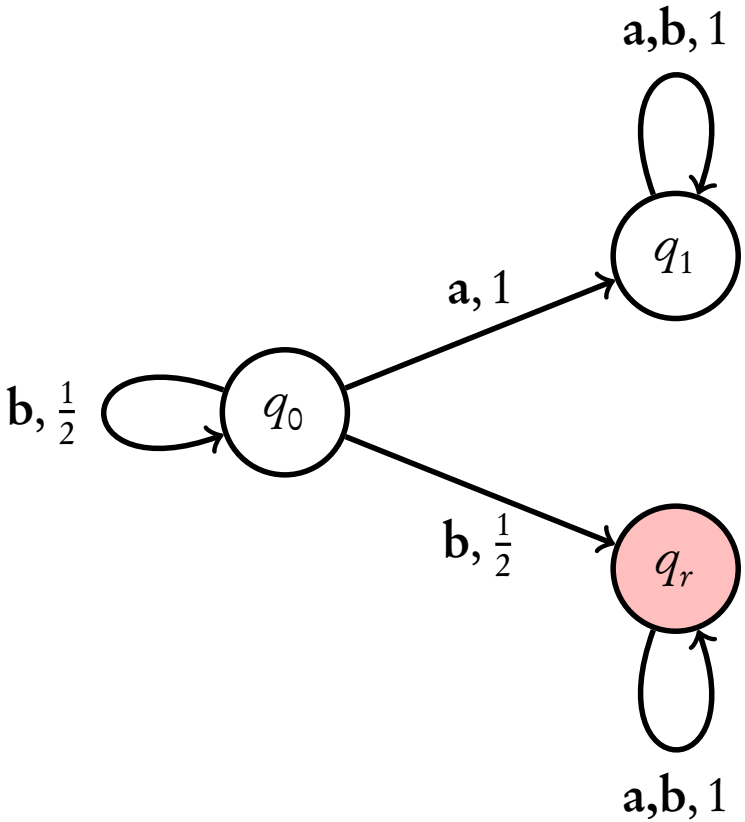
VMCAI 2014

Finite state Probabilistic Monitor [CSV09]



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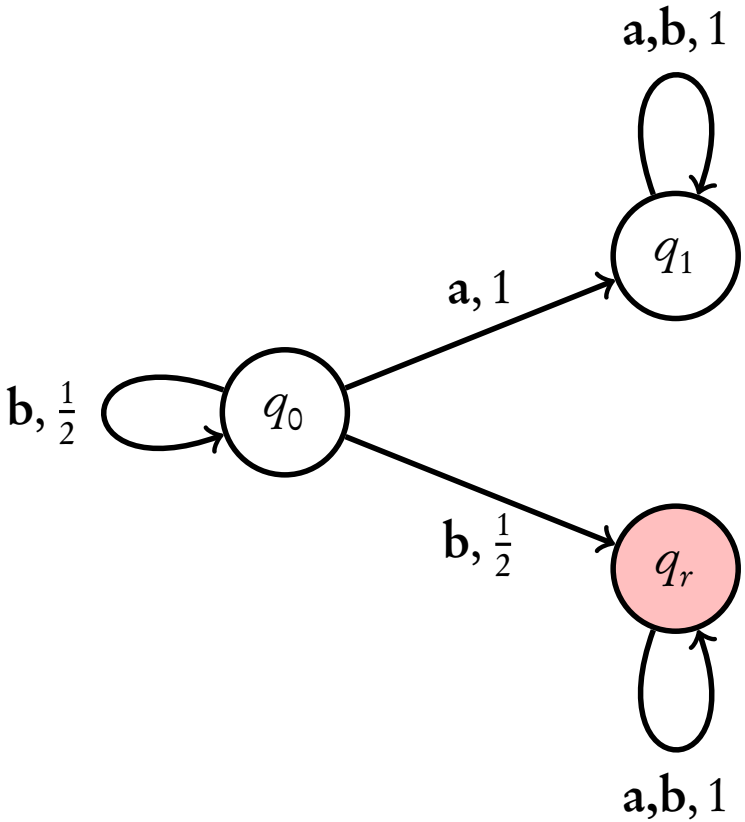
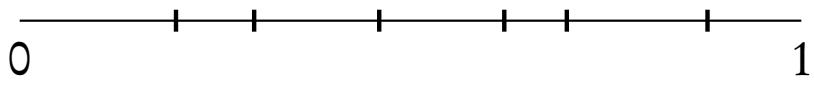


Rejection Probability: Probability of reaching the reject state after having consumed the word.

Monitorable languages



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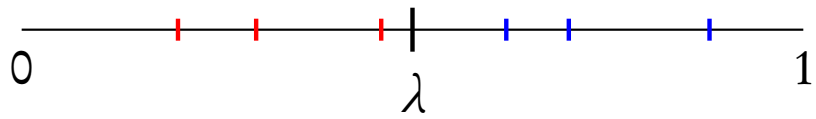

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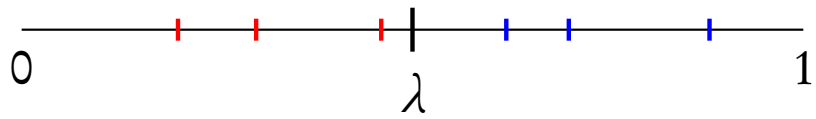

Given a cutpoint $\lambda \in [0, 1]$, words with acceptence probability at least (more than) λ is denoted by $L_{\geq\lambda}(\mathcal{M})$ (resp. $L_{>\lambda}(\mathcal{M})$).

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Language L is monitorable/recognizable

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- **weakly** if there is an $\mathcal{M}$ such that $L_{>0}(\mathcal{M}) = L$.
- **robustly** if there is an $\mathcal{M}$ and λ such that $L_{>\lambda}(\mathcal{M}) = L$ and $\mathcal{M}$ is *isolated* at λ.

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isolated: $\exists\, \epsilon$ such that no word is accepted with probability $(\lambda - \epsilon, \lambda + \epsilon)$.

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Applications

- Optimal FPMs yield efficient monitoring algorithms
- Qualitative verification of Markov Chains [BG05]

Overview

We consider logics Safe-LTL and LTL(G), and investigate bounds on the size of equivalent FPMs

	DBA	NBA	Strong Monitors	Weak Monitors	Robust Monitors
Safe-LTL	2-EXP	EXP	?	?	?
LTL(G)	2-EXP	EXP	?	?	?

Safety LTL

Syntax

Formulae in Safe-LTL are given by:

$$\varphi ::= p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid X\varphi \mid \varphi R \varphi$$

- where p ranges over propositions P.

- Interpreted over $\alpha = \sigma_0\sigma_1\ldots$ where $\sigma_i \in 2^P$
- $[\![\varphi]\!] = \{\alpha \in (2^P)^\omega \mid \alpha \vDash \varphi\}$

Safety LTL

Strong Monitors

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Strong Monitors

Theorem

For every formula φ in Safe-LTL there is a FPM $\mathcal{M}_\varphi$ of size $O(2^{|\varphi|})$ such that $\mathcal{M}_\varphi$ strongly monitors $[\![\varphi]\!]$, i.e. $L_{\geq 1}(\mathcal{M}_\varphi) = [\![\varphi]\!]$.

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NBA constructed from the alternating automata [Var96] for $\neg\phi$, has a single absorbing accept state.

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Complexity of the disjointness function $R_\epsilon^{A \to B}(g_n) = \Omega(2^n)$ [KNR95].

Safety LTL
Weak Monitors

Theorem

There is a family of Safe-LTL formulas $\{\varphi_n\}_{n\in\mathbb{N}}$ where φ_n is of size $O(n^2)$, such that any family of FPMs $\{\mathcal{M}_n\}_{n\in\mathbb{N}}$ that weakly monitors it, $L_{>0}(\mathcal{M}_n) = [\![\varphi_n]\!]$, has size 2^{2^n}.

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- Consider $\varphi_n \in$ Safe-LTL of size $O(n^2)$ which specifies L_n
- $\mathcal{M}$ that weakly monitors L_n can be used to construct a protocol for g_n which costs $\log_2(|\mathcal{M}|)$ bits.

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- Consider $\varphi_n \in$ Safe-LTL of size $O(n^2)$ which specifies L_n
- $\mathcal{M}$ that weakly monitors L_n can be used to construct a protocol for g_n which costs $\log_2(|\mathcal{M}|)$ bits.
- $|\mathcal{M}| = 2^{\Omega(2^n)}$ from $R_\epsilon^{A\to B}(g_n) = \Omega(2^n)$

LTL(G)

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Syntax and Semantics

Formulae in LTL(G) are given by:

$$\varphi ::= p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid X\varphi \mid G\varphi$$

- $G\varphi$ is $\mathit{false}\, R\, \varphi$, holds when φ is true for all suffixes.

LTL(G)
Weak Monitors

Theorem

For every formula φ in LTL(G) there is a FPM $\mathcal{M}_\varphi$ of size $O(2^{|\varphi|})$ such that $\mathcal{M}_\varphi$ weakly monitors $[\![\varphi]\!]$ i.e. $L_{>0}(\mathcal{M}_\varphi) = [\![\varphi]\!]$.

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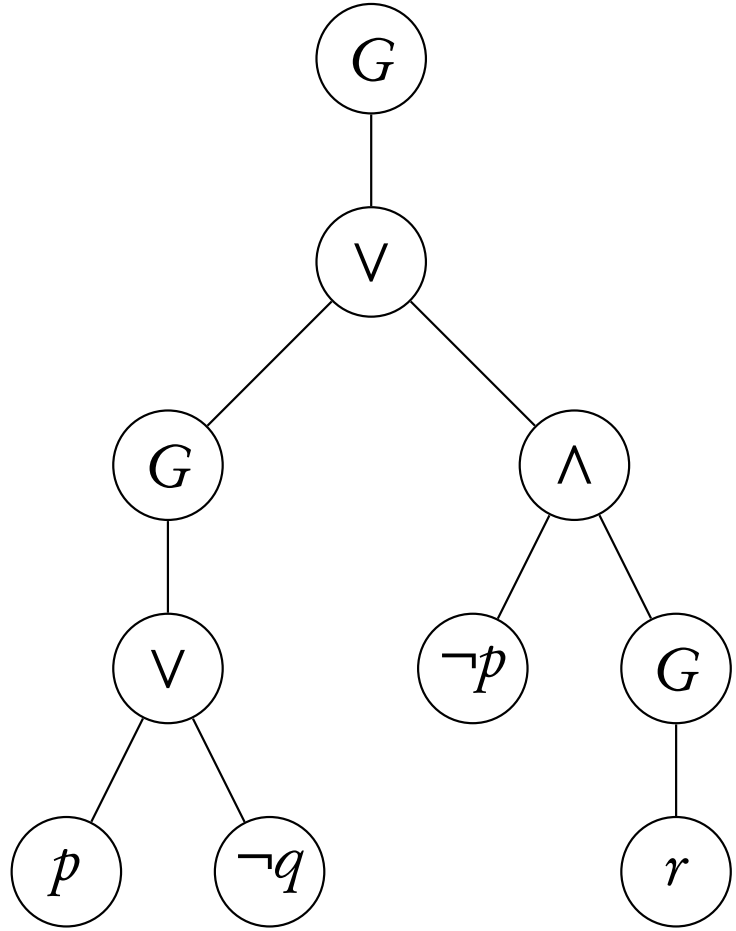
LTL(G)
Weak Monitors



LTL(G)
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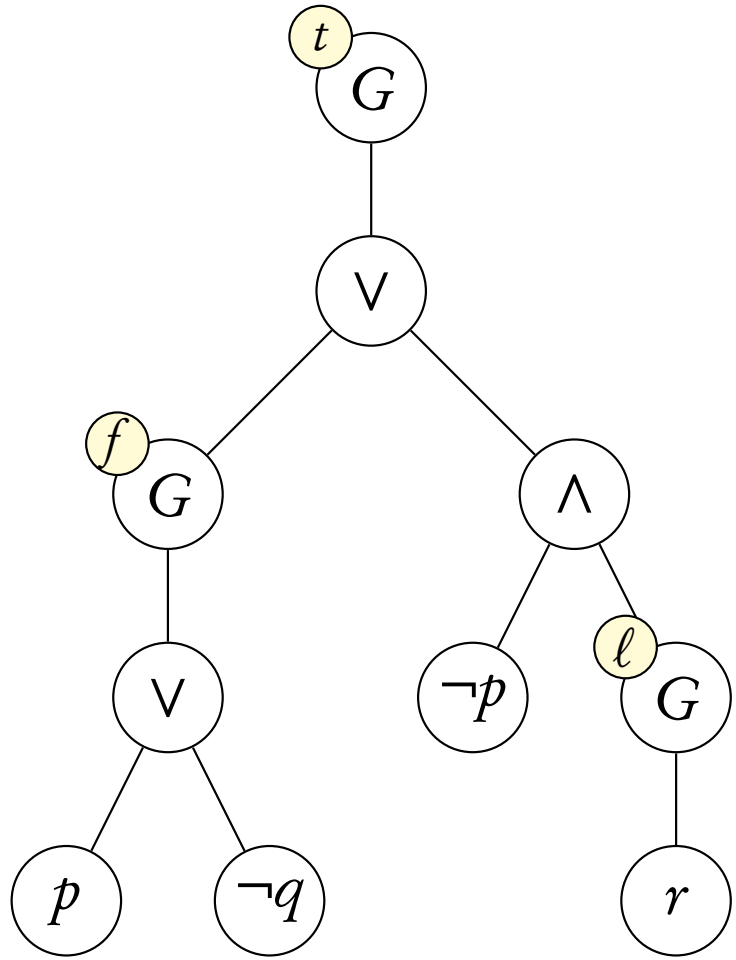
- Annotation: $a : G\psi \to \{t, f, \ell\}$



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- Annotation: $a : G\psi \to \{t, f, \ell\}$
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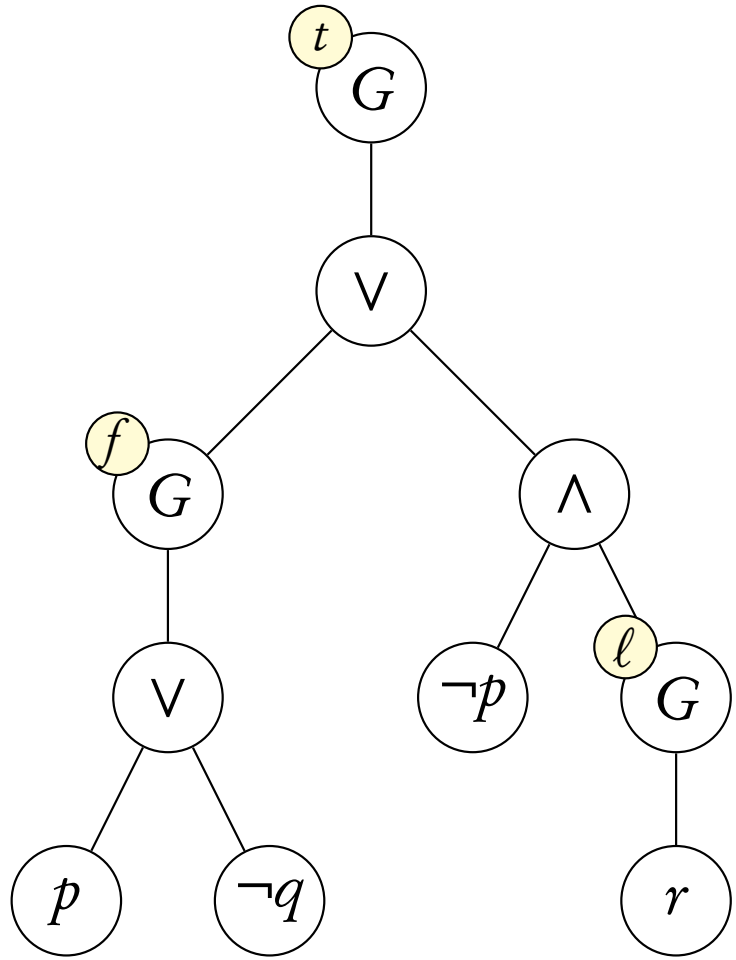

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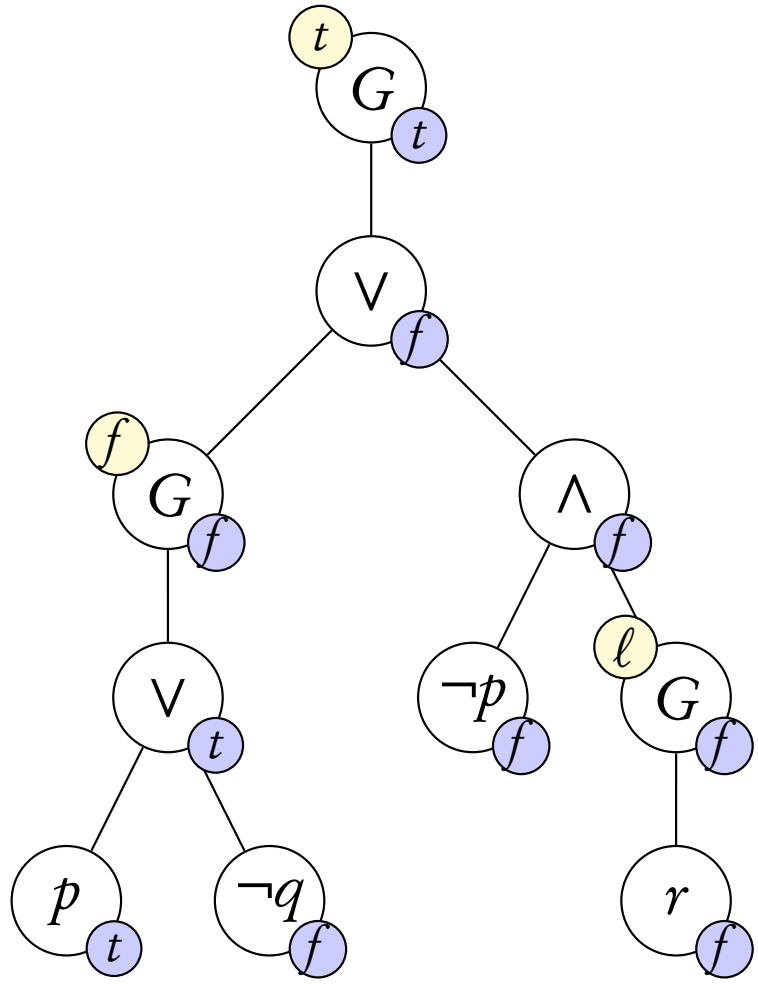
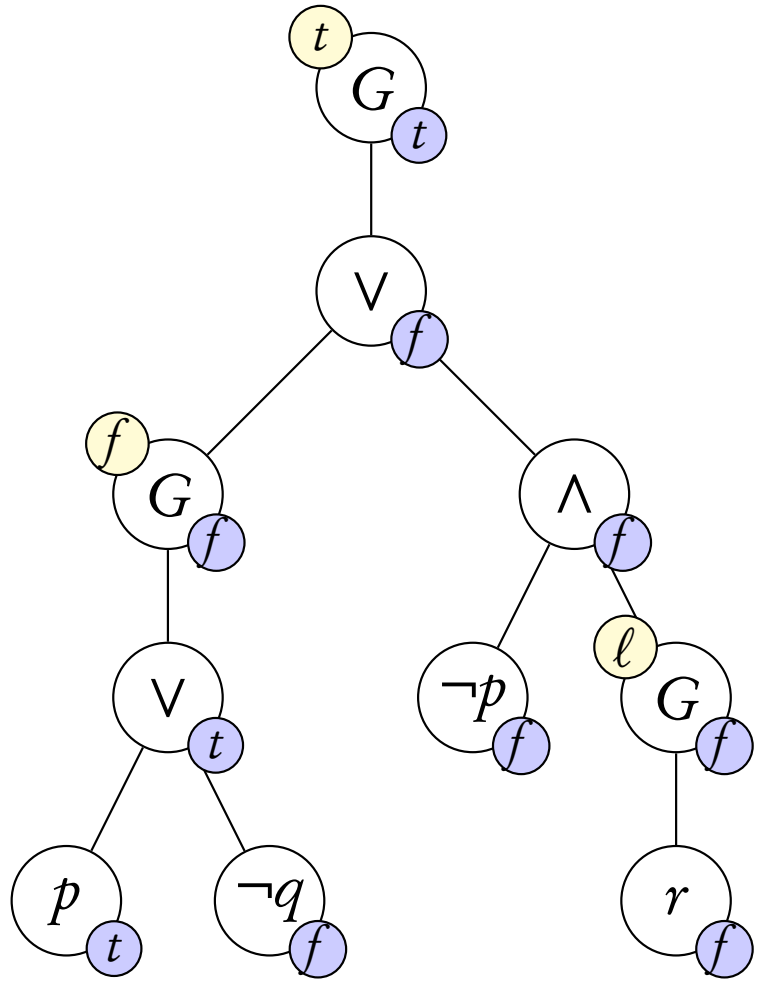


LTL(G)
Weak Monitors

- Annotation: $a : G\psi \to \{t, f, \ell\}$
- Assignment: $\sigma \in 2^P$. eg: $\sigma = \{p, q\}$
- Evaluation: $e^\sigma_a : \psi \to \{t, f\}$

$$a \xrightarrow{\sigma} b$$

- truth of $G\psi$ be preserved from a to b
- if $G\psi$ is true then so should $e^\sigma_a(\psi)$



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Theorem

For every formula $\varphi \in \text{LTL(G)}$ there is a $\mathcal{M}, \lambda$ with $2^{O(|\varphi|)}$ states and $\frac{1}{2^{|\varphi|}}$ gap such that $\mathcal{M}$ with cutpoint λ recognizes φ.

	DBA	NBA	Strong Monitors	Weak Monitors	Robust Monitors
Safe-LTL	2-EXP	EXP	EXP	2-EXP	?
LTL(G)	2-EXP	EXP	EXP	EXP	with gap $\frac{1}{2^n}$: EXP

LTL(G)
Robust Monitors

Key idea: Syntactic Transformation

- Consider $\mathrm{LTL}_{\vee}(\mathrm{G})$ and show every formula has an equivalent *guarded* formula of the same size.
- Every guarded formula has deterministic automata of EXP size
- Convert φ in LTL(G) into a conjunct of $\mathrm{LTL}_{\vee}(\mathrm{G})$ formula by pulling out conjunction.
- Take disjoint union of deterministic monitor for each conjunct and start in any of them with equal probability.

Guarded: every subformula $G(\psi)$ of φ is of the form $G(\alpha \vee \beta)$ where α is disjunction of literals and β is a disjunction of formulae of the form $G\gamma$

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Robust Monitors with Large Gap

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Theorem

There is a family of LTL(G) formulae $\{\varphi_n\}_{n\in\mathbb{N}}$ of size $O(n)$ s.t any family of robust FPM with gap $\frac{1}{2^{o(n)}}$ that recognizes it has size $2^{2^{\Omega(n)}}$.

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Idea: Use communication complexity as before to prove lower bound on constant gap and then use chernoff bounds.

Summary

	DBA	NBA	Strong Monitors	Weak Monitors	Robust Monitors
Safe-LTL	2-EXP	EXP	EXP	2-EXP	with gap $\frac{1}{2^{o(n)}}$: 2-EXP
LTL(G)	2-EXP	EXP	EXP	EXP	with gap $\frac{1}{2^{o(n)}}$: 2-EXP with gap $\frac{1}{2^n}$: EXP

It still remains open whether there are EXP sized robust monitors with small gap for Safe-LTL.

Thank You!

Questions?

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